

Ch 6

Work + Energy

Work done on an object is force applied to an object ~~parallel~~ ~~displacement~~ displacement (magnitude) parallel to its

$$W = F_{\parallel} d$$

Note: scalar value.

or

$$W = Fd \cos \theta$$

θ : angle between force and displacement.

Units: joule

Note: Force w/out work.

Holding groceries? No displacement! $\Rightarrow$ No work.

Moving forward at constant speed?

No force! $\Rightarrow$ No work.

$$\cos \theta = \cos(90^\circ) = 0$$

Ex. 6-1: $m = 50 \text{ kg}$

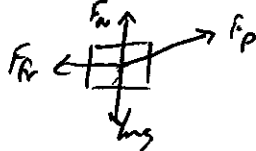
$$d = 40 \text{ m}$$

$$\theta = 37^\circ$$

$$F_p = 100 \text{ N}$$

$$F_f = 50 \text{ N}$$

a.) Work done by each force?



$$W_g = ~~mg~~ mg d \cos 90^\circ = 0$$

$$W_N = F_N d \cos 90^\circ = 0$$

$$W_p = F_p d \cos 37^\circ = (100 \text{ N})(40 \text{ m})(\cos 37^\circ) = 3200 \text{ J}$$

$$W_f = F_f d \cos 180^\circ = (50 \text{ N})(40 \text{ m})(-1) = -2000 \text{ J}$$

Notice: Friction worked opposite the displacement!
Negative work!

Ch. 6

Ex. 6-1 cont...

b.) Net work? (like Net force. but scalars, so just add!)

$$\begin{aligned}W_{\text{net}} &= W_N + W_G + W_P + W_{Fr} \\&= 0 + 0 + 3200\text{J} - 2000\text{J} \\&= 1200\text{J}\end{aligned}$$

Can also be found with net force:

$$F_{\text{net}} = \sum F_x = F_p \cos \theta - F_{Fr}$$

$$W_{\text{net}} = F_{\text{net}} d = (F_p \cos \theta - F_{Fr}) d = 1200\text{J}$$

Ask: Does Earth do positive, negative, or no work on the moon?

Q: 5, 6,

Problems: 5, 8, 9

5. ~~1st: No work~~ ~~2nd: mgh~~ ~~3rd: mg(2h)~~ ~~h = .049 m~~ ~~m = 1.2 kg~~

~~$W_{\text{net}} = mgh(0+1+2+3+4+5+6+7) =$~~

$m = 5.0 \text{ kg}$

$a = 2.0 \text{ m/s}^2$

$t = 7.0\text{s}$

$d = \frac{1}{2} a t^2$

$F = ma$

$W = ma(\frac{1}{2} a t^2) = \boxed{490\text{J}}$

6.]

~~215~~

Ans: 215

$$W_{act} = mgh(0+1+2+\dots+7)$$

a.) $m = M$

a: .10g uprad

$$\Sigma F_y: ma = F - mg$$

$$F = ma + mg = \boxed{Mg(1.10)}$$

$$b) W = Fd \cos \theta = \boxed{Mgh(1.10)}$$

Kinetic Energy

Energy can be described simply as "ability to do work."

~~There are different types of energy; all energy combined is~~

"Total Energy."

"Total Energy" is conserved, or the same before and after a process.

Moving objects have Kinetic Energy

$$\boxed{\text{Kinetic Energy: } KE = \frac{1}{2}mv^2 \quad (\text{Joules})}$$

Object in motion has "ability to do work" If ~~KE changes~~ ^{Net} KE changes, work has been done.

$$W_{NET} = \boxed{\text{KE}_2 - \text{KE}_1}$$

$$\boxed{W_{NET} = \Delta KE} \quad \text{Work energy Principle.}$$

If net work W done on an object is positive, then KE increases.
 If net work W done on an object is negative, then KE decreases.

Ex: 6-4: Baseball being thrown.

$$m = .145 \text{ kg}$$

$$v = 25 \text{ m/s}$$

a.) What is KE? $KE = \frac{1}{2}mv^2 = \boxed{45.5}$

b.) If from rest, what was net work done on the ball?

$$W_{\text{NET}} = \Delta KE = KE_f - KE_i \xrightarrow{0, v=0} = \boxed{45.5}$$

Ex 6-5: Net Work required to accelerate car.

$$m = 1000 \text{ kg}$$

$$v_i = 20 \text{ m/s}$$

$$v_f = 30 \text{ m/s}$$

Net Work? $W = KE_2 - KE_1$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \boxed{2.5 \times 10^5 \text{ J}}$$

Questions: B.) Can KE be negative?

POTENTIAL ENERGY

"ability to do work" as a result of position.

•

~~Simplest~~ Easiest example is a spring.

As you compress a spring, you are "storing up" potential energy into the spring.

Gravitational PE

13

Mass raised to same height has "ability to do work" because of that height, i.e., if dropped, it would do work on whatever it hits.

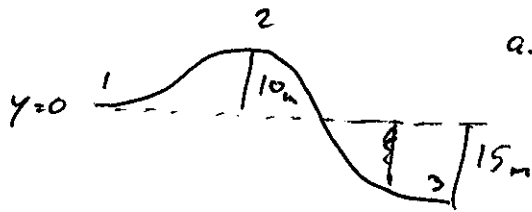
$$\boxed{PE_{\text{grav}} = mgy}$$

Note: depends on our choice of $y = 0$.

Typically, a change in PE gives us important information.

Ex: 6-7: Roller coaster

$$m = 1000 \text{ kg}$$



a.) PE at 2 and 3?

$$PE_2 = mgy_2 = \boxed{9.8 \times 10^4 \text{ J}}$$

$$PE_3 = mgy_3 = \boxed{-1.5 \times 10^5 \text{ J}}$$

b.) Change in PE?

$$PE_3 - PE_2 = \boxed{-2.5 \times 10^5 \text{ J}}$$

Gravitational Potential decreased by $2.5 \times 10^5 \text{ J}$!

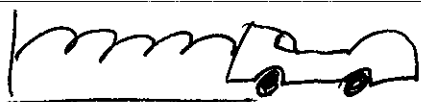
Questions: 14, 17, + B) Can KE be negative?

⑥ Chapt. 6: cont...

11.

Potential Energy

Recall spring + hot wheels example.

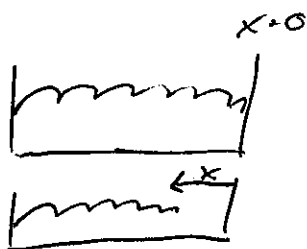


$$F_{\text{spring}} = kx$$

$$PE_{\text{sp}} = \frac{1}{2} kx^2$$

k : spring constant
unique for each spring (like coeff. of friction)

x : amount spring has been stretched/compressed



PROBLEMS: 29, 31, 32

Conservation of Energy:

For now we'll ignore energy lost due to friction, air resistance, etc.
Ignoring those, we can say that Total MECHANICAL ENERGY is conserved!

$$E = KE + PE$$

or

$$KE_f + PE_f = KE_i + PE_i$$

all objects in system!

PE can be grav. or springs/elastic, or both!

38.)

$$\theta = 45.0^\circ$$

$$h = 265 \text{ m}$$

$$V_i = 185 \text{ m/s}$$

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2} m V_i^2 + mgh = \frac{1}{2} m V_f^2 + 0$$

$$V_f^2 = V_i^2 + 2gh$$

$$V_f = \sqrt{V_i^2 + 2gh} = \boxed{199 \text{ m/s}}$$

Test: $V_{yi} = V_i \sin \theta = 130.815 \text{ m/s}$ $V_{xi} \approx 130.815 \text{ m/s}$

$$V_{fy}^2 = V_{iy}^2 + 2g(0-h)$$

$$V_{xf} = 130.815 \text{ m/s}$$

$$V_{fy} = 149.354$$

$$V_f = \sqrt{V_{xf}^2 + V_{yf}^2} = \boxed{199 \text{ m/s}}$$

3

Example: 6.8: Falling rock:

$y_i = 3.0 \text{ m}$ Find rock's V_f .

$y_f = 1.0 \text{ m}$ We could use kinematics!

$$V_f^2 = V_0^2 + 2a(y_0 - y_f)$$

But! let's use ENERGY Conservation!

$$KE_f + PE_f = KE_i + PE_i$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

$$\frac{V_f^2}{2} + gy_f = \frac{1}{2}V_i^2 + gy_i$$

$$V_f^2 = 2g(y_i - y_f)$$

$$V_f = \sqrt{2g(y_i - y_f)} = 6.3 \text{ m/s}$$

Problems: 29, 37, 38

29. $m = 1200 \text{ kg}$
 $V = 65 \times 10^3 \text{ m/h} \cdot \frac{1 \text{ h}}{3600 \text{ s}}$

$x = 2.2 \text{ m}$

$$KE_i + PE_i = KE_f + PE_f$$
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$k = \frac{mv^2}{x^2} = 8.1 \times 10^4 \text{ N/m}$$

37. $m = 65 \text{ kg}$
 $V_i = 5.0 \text{ m/s}$
 $y_i = 3.0 \text{ m}$

a.) $KE_i + PE_i = KE_f + PE_f$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2$$

$$V_f^2 = V_i^2 + 2gy_i \Rightarrow V_f = \sqrt{V_i^2 + 2gy_i} = 9.2 \text{ m/s}$$

b.) $V_f = 0 \Rightarrow KE_i + PE_i = KE_f + PE_f$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}kx^2 \Rightarrow \sqrt{\frac{1}{k}(mv_i^2 + 2mgy_i)} = x =$$

Demos: Kinetic Energy/Work.

$$m = 2.3 \text{ g} = .0023 \text{ kg}$$

$$d = 1.83 \text{ m}$$

$$F = 113 \text{ N due to air pressure}$$

~~113 N~~

How quickly will it accelerate?

$$F = ma \Rightarrow a = \frac{F}{m} \approx 49000 \text{ m/s}^2 \approx \boxed{5000 \text{ g}}!$$

How much work does air pressure do on the ball?

$$W = F_{\parallel} d = F d \approx \boxed{208 \text{ J}}$$

How much Kinetic energy will it have?

$$W = \Delta KE = KE_f - KE_i^{=0} = \boxed{208 \text{ J}} \text{ or } 153 \text{ ft. lbs.}$$

What is its speed at the end of the tube?

$$KE: \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2KE}{m}} = \boxed{425 \text{ m/s}}! \text{ or } \textcircled{1400} \text{ ft/s}$$

950 m/h!

DO DEMO!

Remington gun data:

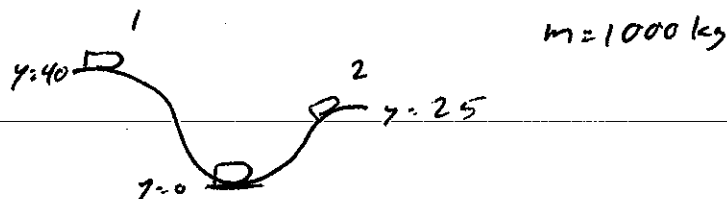
	V ft/s	KE ft-lb	
.25 mm	760	64	← really.
.22 L.R.	1150	117	
.38 S&W	685	150	← Theoretical

Ch. 6 - 9: Dissipative Forces.

Note: Hooke's Law!

$$\boxed{F_s = -kx}$$

Recall Rollercoaster



What is total energy at 1:

$$E = KE + PE = mgh \approx \overset{392000}{400000} \text{ N}$$

What is total energy at 2:

$$E = KE + PE = \frac{1}{2}mv^2 \approx \overset{392000}{400000} \text{ N}$$

What is speed at 3: (v_3)?

$$KE_i + PE_i = KE_f + PE_f$$

$$mgh_1 = \frac{1}{2}mv_3^2 + mgh_3$$

$$g(h_1 - h_3) = \frac{1}{2}v_3^2 \Rightarrow v_3 = \sqrt{2g(h_1 - h_2)} = \boxed{17.1 \text{ m/s}}$$

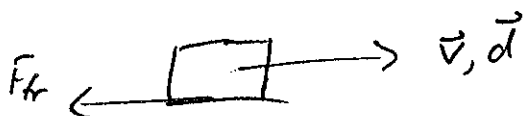
Now: Introduce friction!

Some forces are considered "nonconservative." The ~~work~~ work done by these forces depends on the path taken by the force, and the energy produced by these forces will change KE and PE.

$$\boxed{W_{NC} = \Delta KE + \Delta PE = \Delta E}$$

$$\boxed{KE_i + PE_i = KE_f + PE_f + W_{NC}}$$

Friction is a great example of an NC force.



$$W_{NC} = \vec{F}_{fr} \cdot \vec{d} \cos \theta = -F_{fr} d$$

Ch. 6-9:

Look back at roller coaster. If we say the car travels 400m and the force of friction applied is ~~300~~ 350 N, then what is the speed at point 3?

$$KE_1 + PE_1 = KE_3 + PE_3 + W_{Nc}$$

$$W_{Nc} = -F_k d = -(350\text{N})(400) = -140000\text{N}$$

$$mgh_1 = \frac{1}{2}mv_3^2 + mgh_3 + 140000\text{N}$$

$$mg(h_1 - h_3) + 140000\text{N} = \frac{1}{2}mv_3^2$$

$$\sqrt{2g(h_1 - h_3) + \frac{2}{m}140000\text{N}} = v_3 = 3.7\text{m/s}$$

Same of the energy has been "lost" to friction.
Still conserved; typically is lost as "heat" or "thermal" energy.

Questions: 12, 16, 19

Problems: ~~48~~ 48, ⁵³~~49~~, 51 Hard: 49, 57

Ch. 6-9 Probs

48.) $m = 21.7 \text{ kg}$

$$h = 3.5 \text{ m}$$

$$v_f = 2.2 \text{ m/s}$$

$$\cancel{KE_i + PE_i} = \cancel{KE_f + PE_f} - W_{nc}$$

$$\textcircled{1} mgh = \frac{1}{2}mv_f^2 - W_{nc}$$

$$W_{nc} = \frac{1}{2}mv_f^2 - mgh = \boxed{-692 \text{ J}}$$

53.) $y_1 = 35 \text{ m}$ $v_2?$

$$v_1 = 1.70 \text{ m/s}$$

$$F_f = \frac{1}{5}mg$$

$$d = 45.0 \text{ m}$$

$$W_{nc} = -F_f d = -\frac{1}{5}mgd$$

$$KE_1 + PE_1 = KE_2 + PE_2 - W_{nc}$$

~~cancel~~

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + 0 + F_f d$$

$$\textcircled{2} \frac{1}{2}v_1^2 + gy_1 = \frac{1}{2}v_2^2 + \frac{1}{5}gd$$

$$v_1^2 + 2gy_1 - \frac{2}{5}gd = v_2^2$$

$$v_2 = \sqrt{v_1^2 + 2gy_1 - \frac{2}{5}gd} = \boxed{23 \text{ m/s}}$$

51.) $y_1 = 2.0 \text{ m}$
 $y_2 = 1.5 \text{ m}$

a.) $\frac{\cancel{KE_2} + \cancel{PE_2}}{PE_1} = \frac{|W_{nc}|}{PE_1}$

$$\cancel{KE_1} + PE_1 = \cancel{KE_2} + PE_2 - W_{nc}$$

$$PE_1 - PE_2 = -W_{nc} = mg(.5)$$

$$|W_{nc} = -mg(.5)|$$

$$\frac{mg(.5)}{mg(2.0)} = \boxed{\frac{1}{4}} \quad \boxed{25\%}$$

b.) $\underset{0}{\cancel{KE_1}} + PE_1 = \underset{0}{\cancel{KE_2}} + PE_2 - W_{nc}$

$$mgy_1 + \overset{+W_{nc}}{\cancel{PE_2}} = \frac{1}{2}mv_2^2 \Rightarrow mgy_1 - mg(.5) = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gy_1 + g} = \boxed{5.4 \text{ m/s}}$$

c.) Where did E go?

heat, sound, deformation...

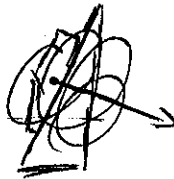
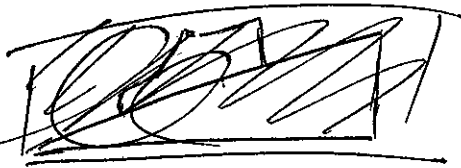
Forces:

3.)

$$m = 65 \text{ kg}$$

$$F_k = 720 \text{ N}$$

$$\theta = 22^\circ$$

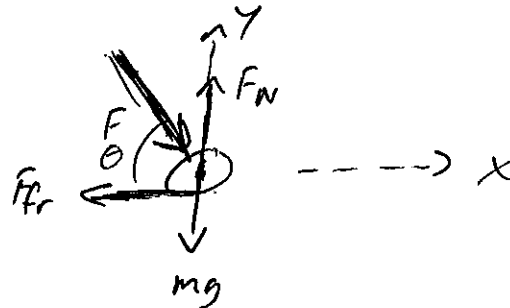


25.) $m = 14.0 \text{ kg}$ a.) Draw FBD.

$$F = 88.0 \text{ N}$$

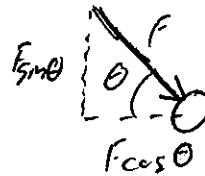
$$\theta = 45.0^\circ$$

$$a = 0$$



$$c + b.) \cdot \Sigma F_y = ma_y = 0$$

$$\Sigma F_y = F_N - mg - F \sin \theta$$



$$F_N = mg + F \sin \theta = \boxed{199 \text{ N}}$$

$$\Sigma F_x = ma_x = 0$$

$$\Sigma F_x = F \cos \theta - F_{fr}$$

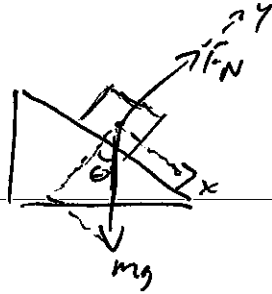
$$F \cos \theta = F_{fr} = \boxed{62 \text{ N}}$$

$$d.) a_x = \frac{V_k - V_{ox}}{t} = \frac{1.5}{2.5} = .6 \text{ m/s}^2$$

$$F \cos \theta - F_{fr} = ma_x \Rightarrow \boxed{F = \frac{ma_x + F_{fr}}{\cos \theta} = 100 \text{ N}}$$

53. $V_0 = 3.0 \text{ m/s}$ No friction

$\theta = 22.0^\circ$



$$\Sigma F_x = m a_x = m g \sin \theta$$

$$\Sigma F_y = m a_y = 0: F_N - m g \cos \theta$$

$$a_x = g \sin \theta$$

Work like free fall:

$$0 = V^2 = V_0^2 + 2 a_x (x - x_0)$$

$$0 = V_0^2 + 2 a_x x \Rightarrow$$

$$x = \frac{-V_0^2}{2 g \sin \theta} = -1.2 \text{ m}$$

$$0 = V_0 + a_x t \Rightarrow -V_0 = a_x t$$

$$t = \frac{-V_0}{g \sin \theta} = 1.6 \text{ s}$$

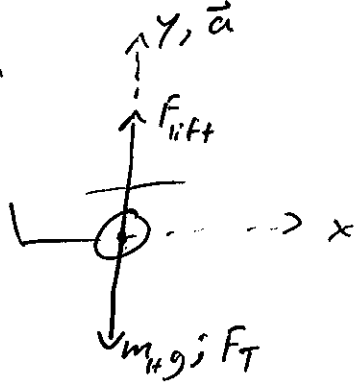
$$x = x_0 + V_0 t + \frac{1}{2} g \sin \theta t^2$$

$$0 = 0$$

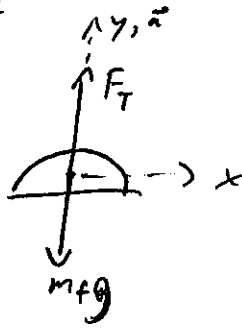
$$-V_0 t = \frac{1}{2} g \sin \theta t^2 \Rightarrow$$

$$t = \frac{2 V_0}{g \sin \theta} = 1.6 \text{ s}$$

Pl.) $m_H = 7650 \text{ kg}$ FBD: heli
 $a_y = 0.80 \text{ m/s}^2$
 $m_F = 1250 \text{ kg}$



FBD: Frame



a.)

Heli: $\sum F_y = m_H a_y$
 $\sum F_y = F_{lift} - m_H g - F_T$

$$m_H a_y = F_{lift} - m_H g - F_T$$

$$\Rightarrow \boxed{F_{lift} = m_H a_y + m_H g + F_T}$$

FRAME: $\sum F_y = m_F a_y$

$$\sum F_y = F_T - m_F g$$

$$\boxed{F_T = m_F a_y + m_F g}$$

$$\Rightarrow F_{lift} = m_H a_y + m_H g + m_F a_y + m_F g = \boxed{94300 \text{ N}}$$

b.)

$$\Rightarrow F_T = \cancel{13300} \boxed{13300 \text{ N}}$$

c.)

$$\boxed{F_T = 13300 \text{ N}}$$

Ch. 6: Circular motion

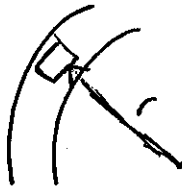
9.) $m = 1050 \text{ kg}$

$r = 77 \text{ m}$

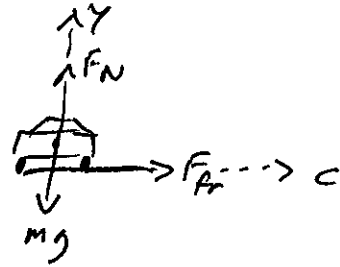
$\mu_s = 0.80$

$V = ?$

Top:



Side



$$\Sigma F_c = m a_c = m \frac{v^2}{r}$$

$$\Sigma F_c = F_{fr} = \mu_s F_N \rightarrow m \frac{v^2}{r} = \mu_s F_N$$

$$\Sigma F_y = m a_y = 0 \Rightarrow |F_N = mg|$$

$$\frac{mv^2}{r} = \mu_s mg \Rightarrow \frac{v^2}{r} = \mu_s g$$

$$V = \sqrt{\mu_s g r} = 25 \text{ m/s}$$

b.) yes.

12.] $r = .11 \text{ m}$

ANSWER: 0.16

~~$f = 36 \text{ rpm} = 36 \cdot \frac{2\pi}{60}$~~

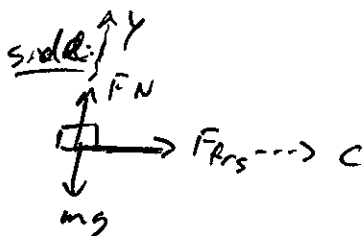
~~$f = 36 \text{ rpm} = 36 \cdot \frac{2\pi}{60}$~~

$f = 36 \frac{\text{rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = .6 \frac{\text{rev}}{\text{s}}$

~~$f = \frac{1}{T}$~~

$v = \frac{2\pi r}{T} = 2\pi r f = .415 \text{ m/s}$

Find μ_s .



$\sum F_c = m \frac{v^2}{r} = F_{frs}$

Note: $F_{frs} = \mu_s F_N$

$\Rightarrow m \frac{v^2}{r} = \mu_s F_N$

$\sum F_y = m a_y = 0$

$\sum F_y = F_N - mg \Rightarrow F_N = mg$

So: $\frac{mv^2}{r} = \mu_s mg \Rightarrow$ ~~$\mu_s = \frac{v^2}{gr}$~~

$\mu_s = \frac{v^2}{gr} = .16$

Ch. 5

Q. 17: What rpm?

$$r = .09 \text{ m}$$

$$a_c = 115000 \text{ g}$$

$$a_c = \frac{v^2}{r} \Rightarrow v = \sqrt{a_c r}$$

$$\text{Note: } v = \frac{2\pi r}{T} = 2\pi r f$$

$$2\pi r f = \sqrt{a_c r}$$

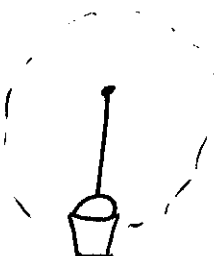
$$f = \frac{\sqrt{a_c r}}{2\pi r} = 563.2 \frac{\text{rev}}{\text{s}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = \boxed{33792 \text{ rpm}}$$

ANS: a.) 1.72 m/s b.) 3.3 m/s

16. $m = 2.00 \text{ kg}$

$$r = 1.10 \text{ m}$$

$$F_{T \text{ low}} = 25.0 \text{ N}$$



FBD:



$$\Sigma F_c = m \frac{v^2}{r}$$

$$\Sigma F_c = F_T - mg$$

$$\frac{m v^2}{r} = F_T - mg \Rightarrow v^2 = \frac{r}{m} (F_T - mg)$$

$$v = \sqrt{\frac{r}{m} (F_T - mg)} = \boxed{1.72 \text{ m/s}}$$

AT TOP: $F_T \rightarrow 0$ just before rope goes slack



$$\Sigma F_c = m \frac{v^2}{r}$$

$$\Sigma F_c = F_T + mg \Rightarrow \Sigma F_c = mg \text{ (at top)} \rightarrow \frac{m v^2}{r} = mg \Rightarrow \boxed{v = \sqrt{gr} = 3.3 \text{ m/s}}$$

Ch. 6 Review

Work: $W = F_{||} d = F d \cos \theta$

Net Work: $W_{NET} = W_1 + W_2 + W_3 + W_4 + \dots$

Where ~~the~~ W_1 is the work done by some "force 1."

KINETIC ENERGY: $KE = \frac{1}{2} m v^2$

WORK-ENERGY PRINCIPLE: $W_{NET} = \Delta KE = KE_f - KE_i$

POTENTIAL ENERGY due to Gravity: $PE_{grav} = mgy$

Hooke's Law (Spring force): $F_{spring} = -kx$

POTENTIAL ENERGY due to Spring: $PE_{sp} = \frac{1}{2} k x^2$

CONSERVATION OF ENERGY:

without friction: $KE_1 + PE_1 = KE_2 + PE_2$

with friction: $KE_1 + PE_1 = KE_2 + PE_2 - W_{nc}$

where: $W_{nc} = -F_{fr} d$

QUESTIONS: 7, 11, 18, ~~20~~

Ch. 6: POWER

Power: rate at which work is done.

$$\boxed{P = \frac{W}{t}} \quad \text{units: Watt, W or } \frac{J}{s}$$

Ex: 6-14: $m = 60 \text{ kg}$

$t = 4.0 \text{ s}$

$d = 4.5 \text{ m}$

a.) Power output in Watts & hp?

$$P = \frac{W}{t}$$

$$W = Fd \cos \theta = mgd$$

$$\Rightarrow P = \frac{mgd}{t} = \boxed{660 \text{ W}} \cdot \frac{1 \text{ hp}}{746 \text{ W}} = \dots$$

b.) How much energy?

$$\text{Energy} = \text{Power} \cdot \text{time}$$

$$E = \bar{P}t = 660 \cdot t = 2600 \text{ J}$$

As expected, this equals $W = mgd$.

Ch. 6: Power continued...

Note:

$$P = \frac{W}{t} = \frac{Fd}{t} = F \frac{d}{t} = F v_{avg}$$

~~Q. 24,~~ Q. 24,

Problems:

58. $P = \frac{W}{t} \Rightarrow W = Fd \Rightarrow t = \frac{Fd}{P}$

59. $P = F v_{avg} \Rightarrow F = \frac{P}{v_{avg}}$

60. $m = 1400 \text{ kg}$

$a = \frac{95 \text{ km/h}}{7.4 \text{ s}} \Rightarrow x = \cancel{v_0 t} + \cancel{v_0 t} + \frac{1}{2} a t^2$

$$F = ma$$

$$W = Fd = max$$

$$P = \frac{W}{t} = \frac{max}{t}$$

Questions: 24.

Problem: 83, 81, 93

83.) a.) $m = 82 \text{ kg}$

$$E = 1100 \text{ kJ} = 1100 \times 10^3 \text{ J}$$

$$E = \cancel{KE} + PE = mgh$$

$$h = \frac{E}{mg} = \boxed{1.4 \times 10^3 \text{ m}}$$

b.) Speed when jump?

$$\cancel{KE_i} + PE_i = \cancel{KE_f} + \cancel{PE_f}$$

$$\cancel{mgh} = \frac{1}{2} \cancel{mv^2}$$

$$v = \sqrt{2gh} = \boxed{166 \text{ m/s}}$$

81.) $m = 650 \text{ kg}$ in 1 s ; $t = 1 \text{ s}$
 $h = 81 \text{ m}$

a.) $v = \sqrt{2gh} = \boxed{40 \text{ m/s}}$

b.) $P = \frac{W}{t} = \cancel{\frac{mgh}{t}} = \frac{\Delta KE}{t} = \frac{\frac{1}{2}mv^2}{t}$

$$\text{Actual } P = .58P = .58 \cdot \left(\frac{.5mv^2}{t} \right) = (.58)(.5)mv^2 = (.58) \cancel{mgh} =$$

$$\boxed{P = 300,000 \text{ W}}$$

Per U.S. House: ~~$4.42 \times 10^6 \text{ W}$~~

$$\frac{4.42 \times 10^6 \text{ W}}{112,000,000} \approx 4000 \text{ W} \Rightarrow \boxed{75 \text{ homes!}}$$

93. a.)

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} \approx \boxed{100 \text{ m/s}}$$

b.)

$$P = \frac{W}{t} = \frac{\Delta KE}{t} = \frac{mgh \cdot 1000}{60} = \boxed{4 \times 10^7 \text{ W}}$$

$$\frac{4 \times 10^7 \text{ W}}{4000 \text{ W}} \approx \boxed{10000 \text{ Homes!}}$$

Chapter 7. | Linear Momentum.

Today we introduce a new, important quantity called momentum.

Momentum is similar to energy in that it is conserved.

Unlike energy, Momentum is a vector, so it will have magnitude and direction, which, unfortunately, means components.

~~Chapter~~ This chapter focuses on Linear Momentum.

momentum = mass times velocity

$$\vec{p} = m \vec{v}$$

Components: $p_x = m v_x$

$$p_y = m v_y$$

$$p_z = m v_z$$

Example: 4.5 kg meteoroid is traveling through space at 10.7 m/s.

It moves in an x-y plane with an angle of 72° measured clockwise from positive y.

- a) What are the x and y components of linear momentum? $\frac{x}{46 \frac{\text{kg} \cdot \text{m}}{\text{s}}}$, $\frac{y}{15 \frac{\text{kg} \cdot \text{m}}{\text{s}}}$
- b) What is magnitude and direction of linear momentum? $48 \frac{\text{kg} \cdot \text{m}}{\text{s}}$, in line w/ velocity

Ch. 7

Momentum and Force

Newton's second law: $\Sigma F = ma$

was originally written in terms of momentum: $\Sigma F = \frac{\Delta p}{\Delta t}$

$$\Sigma F = ma = m \frac{\Delta v}{\Delta t} = \frac{mv_2 - mv_1}{\Delta t} = \frac{\Delta p}{\Delta t}$$

Ex. 7.1: Tennis Serve: What is the average force on the ball?

$$v_f = 55 \text{ m/s} \quad v_o = 0 \text{ m/s}$$

$$t = 4 \text{ ms} = 4 \times 10^{-3} \text{ s}$$

$$m = 0.060 \text{ kg}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_f - mv_o}{\Delta t} = \frac{mv_f}{t} \approx \boxed{800 \text{ N}}$$

Could this lift a 60-kg (~132 lb) person?

$$(60 \text{ kg})(g) \approx \boxed{600 \text{ N. Yes!}}$$

Questions: 10, 7, 6

Problems: 3, 9, ~~20, 21~~ (1 + 2)

$$3.) m = 0.145 \text{ kg}$$

$$v_o = -39.0 \text{ m/s}$$

$$v_f = 52.0 \text{ m/s}$$

$$t = 3.00 \times 10^{-3} \text{ s}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_o)}{t}$$

$$\boxed{4400 \text{ N toward pitcher}}$$

9. $V_0 = 100 \text{ km/h}$

$F_{\text{wind}}?$

$\frac{m}{A} = 40 \text{ kg/m}^2$

$t = 1 \text{ s}$

$A = 1.50 \text{ m} \times 0.50 \text{ m} = .75 \text{ m}^2$

$$F = \frac{\Delta p}{\Delta t} = \frac{m V_f - m V_0}{t}$$

$V_f = 0$

$m = \frac{m}{A} \cdot A = 30 \text{ kg}$

$V = 100 \frac{\text{km}}{\text{h}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = \cancel{60000} \text{ m/s}$

$m V_0 = \cancel{150000} \frac{\text{kg} \cdot \text{m}}{\text{s}}$

$F \approx \cancel{150000} \text{ N} \approx 833 \text{ N} \approx \boxed{800 \text{ N}}$

$F_{fr} = \mu F_N = \mu m g \approx \boxed{700 \text{ N}}$

1. $p = mv$

$m = .028 \text{ kg}$

$v = 8.4 \text{ m/s}$

2. $F = 25 \text{ N}$

$m = 65 \text{ kg}$

$t = 2 \text{ s}$

$$F = m \frac{\Delta v}{\Delta t}$$

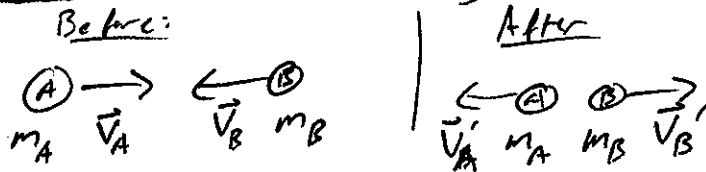
$$\Delta v = \frac{F \Delta t}{m}$$

Ch. 7 | Conservation of Momentum: Collisions.

Like energy, Momentum is conserved

That is: $\vec{p}_f = \vec{p}_i$ if we have no external forces acting

~~Consider~~ Consider two colliding billiard balls, A + B.



We can say, due to conservation, that:

$$\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$$

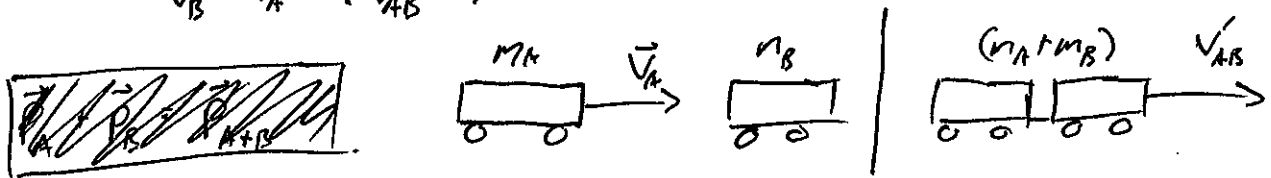
or

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B$$

Note: $v_A \neq v'_A$
 $v_B \neq v'_B$

~~the net force is zero, the net force~~

Example 7-3: $m_A = 10,000 \text{ kg}$ $m_B = 10,000 \text{ kg}$
 $v_A = 24.0 \text{ m/s}$ $v_B = 0 \text{ m/s}$
 $v'_B = v'_A = v'_{AB} = ?$



$$\vec{p}_A + \vec{p}_B = \vec{p}'_{AB} \quad \text{only x} \Rightarrow p_A + p_B = p'_{AB}$$

$$p_A = p'_{AB}$$

$$m_A v_A = (m_A + m_B) v'_{AB}$$

$$v'_{AB} = \left(\frac{m_A}{m_A + m_B} \right) v_A = \left(\frac{10000}{20000} \right) (24) = \boxed{12.0 \text{ m/s}}$$

So, there was more mass moving after the collision, so the ~~velocity~~ speed was slower.

What if there was less mass moving after the collision?

~~Q2.10~~

Supernova process: Type II, "Core Collapse" Supernova.

- Star runs out of fuel at core
- Begins fusion, turning Helium $\rightarrow$ Carbon, Carbon $\rightarrow$ Neon, etc.
- eventually stops at iron, and all the matter collapses at about 70,000 km/s.
- Temperatures rise to 100 billion Kelvin ($6000 \times T_{\text{sun}}$ core)
- Collapsing matter rebounds off the core and...

DEMO:

This is how all of the heavier elements ~~in~~ ^{in the} Universe are ~~created even the ones on Earth~~ created and get spread across the universe.

Questions: 2, 3, 4

Problems: 11 (can't convert mass unit, just use "u")

23 Use KE, also!

25 "

Note: "Elastic" tells you that the objects don't stick together. AND KINETIC ENERGY IS CONSERVED!

Note: Impulse

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} \Rightarrow \vec{F} \Delta t = \Delta \vec{p} = \text{Impulse}$$

Ex: ball of mass .130 kg hit by bat. off of a tee-ball tee.
 Ball leaves the bat with a velocity of 8.7 m/s in x-direction.
 Time interval of contact was 0.024 s.

a.) p_i and p_f ? $p_i = 0 \frac{\text{kg m}}{\text{s}}$; $\vec{p}_f = m \vec{v}_f = \boxed{1.1 \frac{\text{kg m}}{\text{s}} (x)}$

b.) What is impulse delivered?

Impulse: $\Delta \vec{p} = \boxed{1.1 \text{ kg m/s } (x)}$

c.) Average force exerted?

$$\Delta \vec{p} = \vec{F} \Delta t \Rightarrow \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \boxed{46 \text{ N } (x)}$$

Collisions:

2 Types: Elastic and Inelastic

Elastic: Momentum AND KINETIC ENERGY conserved

$$\vec{p}_{i \text{ Total}} = \vec{p}_{f \text{ Total}}$$

$$KE_{i \text{ Total}} = KE_{f \text{ Total}}$$

Cons. of KE + Cons. of ~~KE~~ Mom.

Mom.:

$$\boxed{m_A V_A + m_B V_B = m_A V'_A + m_B V'_B}$$

KE:

$$\frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 = \frac{1}{2} m_B V_B'^2 + \frac{1}{2} m_A V_A'^2$$

Rewriting: Mom.:

$$m_A (V_A - V'_A) = m_B (V_B - V'_B) \quad (i)$$

KE:

$$m_A (V_A^2 - V_A'^2) = m_B (V_B^2 - V_B'^2)$$

Recall: $(a^2 - b^2) = (a - b)(a + b)$

Then:

$$m_A (V_A - V'_A) (V_A + V'_A) = m_B (V_B - V'_B) (V_B + V'_B) \quad (ii)$$

Divide! (ii) by (i)

$$V_A + V'_A = V_B + V'_B$$

Rewrite:

$$\boxed{V_A - V_B = -(V'_A - V'_B)}$$

Cons. of KE. for 2 obj. in 1 dimension!
Elastic!

Ex. 7-7

Ball A collides with stationary ball B of equal mass.
Assuming elastic collision, what are the speeds afterwards?

$$V_A = V \quad V_A' = ? \quad m_A = m_B = m$$

$$V_B = 0 \quad V_B' = ?$$

Conservation of Momentum

$$\vec{P}_{\text{total}} = \vec{P}_{\text{total}}$$

$$mV_A + mV_B = mV_A' + mV_B'$$

$$V_A = V = V_A' + V_B'$$

Conservation of KE

$$\frac{1}{2}mV_A^2 + \frac{1}{2}mV_B^2 = \frac{1}{2}mV_A'^2 + \frac{1}{2}mV_B'^2$$

$$\cancel{\frac{1}{2}mV_A^2} + \cancel{\frac{1}{2}mV_B^2} = \cancel{\frac{1}{2}mV_A'^2} + \cancel{\frac{1}{2}mV_B'^2}$$

Recall:

$$V_A - V_B = -(V_A' - V_B')$$

or here:

$$V = V_B' - V_A'$$

2 Equations!

2 Unknowns!

Combine:

$$V = V_A' + V_B' = V_B' - V_A'$$

$$\Rightarrow V_A' + V_A' = V_B' - V_B'$$

$$\Rightarrow 2V_A' = 0$$

$$\boxed{V_A' = 0}$$

Plug in:

$$V = V_B' - V_A' \Rightarrow \boxed{V_B' = V}$$

This is shown
in picture
7-15

Problems: 23, 25, 27

23.) $m_A = 0.450 \text{ kg}$ $V_A = 3.00 \text{ m/s}$
 $m_B = 0.900 \text{ kg}$ $V_B = 0 \text{ m/s}$

$$m_A V_A = m_A V_A' + m_B V_B'$$

$$\Rightarrow \boxed{V_A = V_A' + \frac{m_B V_B'}{m_A}}$$

$$V_A - V_B = -(V_A' - V_B')$$

$$\Rightarrow \boxed{V_A = V_B' - V_A'}$$

Combine:

$$V_A' + \frac{m_B V_B'}{m_A} = V_B' - V_A' \Rightarrow 2V_A' = V_B' \left(1 + \frac{m_B}{m_A}\right)$$

$$V_A' = \frac{1}{2} V_B' \left(1 + \frac{m_B}{m_A}\right) \approx \overset{-5}{\cancel{0.5}} V_B'$$

$$V_A' = \overset{-5}{\cancel{0.5}} V_B'$$

Plug in:

$$V_A = V_B' - (\overset{-5}{\cancel{0.5}} V_B') = V_B' (1.5)$$

$$V_B' = \frac{V_A}{1.5} = \boxed{+2.00 \text{ m/s east}}$$

$$V_A' = V_B' - V_A = \boxed{-1.00 \text{ m/s west}}$$

25. | $m_A = 0.060 \text{ kg}$ $m_B = 0.090 \text{ kg}$
 $V_A = 2.50 \text{ m/s}$ $V_B = 1.15 \text{ m/s}$ (~~2.50 m/s~~)

Speed/direction after?

Momentum:

$$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$$

Energy:

$$V_A - V_B = -(V_A' - V_B') \Rightarrow V_A - V_B = V_B' - V_A' \Rightarrow \boxed{V_B' = V_A - V_B + V_A'}$$

Plug in:

$$m_A V_A + m_B V_B = m_A V_A' + m_B V_A' - m_B V_B + m_B V_A'$$

$$\Rightarrow \frac{m_A}{m_B} V_A + V_B = \frac{m_A}{m_B} V_A' + V_A - V_B + V_A'$$

$$\Rightarrow V_A \left(\frac{m_A}{m_B} - 1 \right) + 2V_B = V_A' \left(\frac{m_A}{m_B} + 1 \right)$$

$$V_A' = \frac{V_A \left(\frac{m_A}{m_B} - 1 \right) + 2V_B}{\left(\frac{m_A}{m_B} + 1 \right)} = \boxed{0.88 \text{ m/s}}$$

$$V_B' = V_A - V_B + V_A' = \boxed{2.23 \text{ m/s}}$$

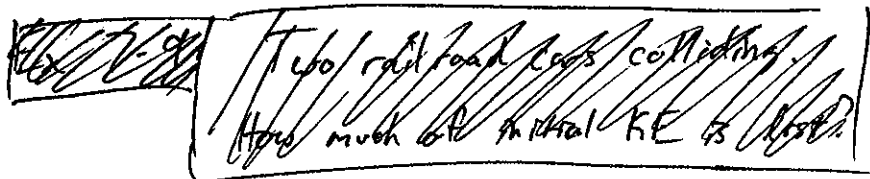
Ch. 7

Inelastic Collisions

Kinetic Energy is NOT conserved here.

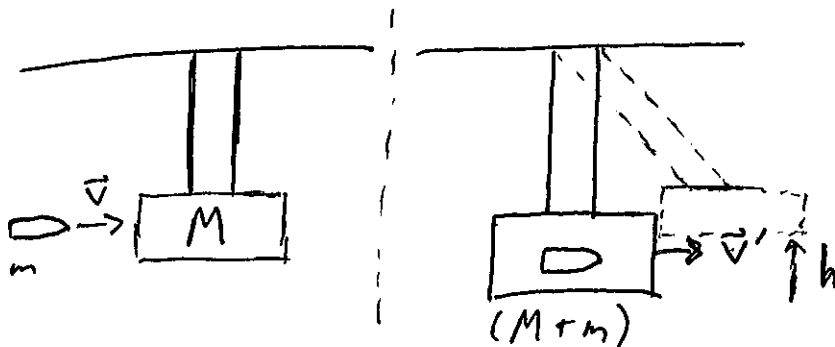
But! Momentum still is!

If two objects stick together, it is then completely inelastic.



Ex. 7-10: Ballistic Pendulum

Bullet of mass m is shot into large block of mass M .



Find a relationship between $\vec{v}$ and h .

1.) Cons. of momentum:

$$\vec{p}_{\text{total}} = \vec{p}'_{\text{total}} \Rightarrow m\vec{v} + \cancel{M\vec{v}_m} = (M+m)\vec{v}' \Rightarrow \boxed{m\vec{v} = (M+m)\vec{v}'}$$

2.) Cons. of Energy (after collision)

$$KE_i + PE_i = KE_f + PE_f \Rightarrow \frac{1}{2}(M+m)\vec{v}'^2 + \cancel{(M+m)g \cdot 0} = \frac{1}{2}(M+m)\vec{v}_f^2 + (M+m)gh$$
$$\Rightarrow \frac{1}{2}\cancel{(M+m)}\vec{v}'^2 = \cancel{(M+m)}gh \Rightarrow \boxed{v' = \sqrt{2gh}}$$

Ch. 7

So we have: $m\vec{v} = (M+m)\vec{v}'$
and $\vec{v}' = \sqrt{2gh}$

~~Calculus~~ Plug-in $\vec{v}'$:

$$m\vec{v} = (M+m)\sqrt{2gh}$$

$$\boxed{v = \frac{(M+m)}{m} \sqrt{2gh}}$$

Problems, Front room: #32, 35, 31

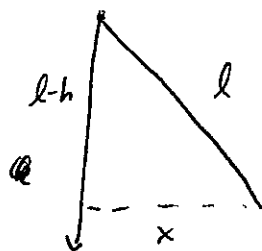
#32. $h_1 = .026 \text{ m}$
 $h_2 = .052 \text{ m}$

$$v = \frac{M+m}{m} \sqrt{2gh}$$

$$\frac{v_2}{v_1} = \frac{\left(\frac{M+m}{m}\right) \sqrt{2gh_2}}{\left(\frac{M+m}{m}\right) \sqrt{2gh_1}} = \sqrt{\frac{h_2}{h_1}} = \boxed{1.4}$$

#32. $m = .028 \text{ kg}$
 $v = 230 \text{ m/s}$
 $M = 3.6 \text{ kg}$
 $l = 2.8 \text{ m}$

$$h = v^2 \left(\frac{m}{M+m} \right)^2 \cdot \frac{1}{2g} = .16 \text{ m}$$



$$x = \sqrt{l^2 - (l-h)^2}$$

#35 $m_1 = 920 \text{ kg}$
 $m_2 = 2300 \text{ kg}$
 $d = 2.8 \text{ m}$
 $\mu = 0.80$

$$\frac{1}{2}(m_1+m_2)V_0^2 = F_{fr}d = \mu(m_1+m_2)gd$$

$$m_1 V = (m_1+m_2)V_0$$

$$\boxed{V_0 = \left(\frac{m_1}{m_1+m_2} \right) V}$$

$$\frac{1}{2}(m_1+m_2)V_0^2 = F_{fr}d = \mu(m_1+m_2)gd$$

$$\frac{1}{2}V_0^2 = \mu gd$$

$$\boxed{V_0 = \sqrt{2\mu gd}}$$

$$\left(\frac{m_1}{m_1+m_2} \right) V = \sqrt{2\mu gd} \Rightarrow V = \frac{(m_1+m_2)}{m_1} \sqrt{2\mu gd} = \boxed{23.1 \text{ m/s}}$$

27.) $m_A = 450 \text{ kg}$ $v_A = 4.50 \text{ m/s}$
 $m_B = 550 \text{ kg}$ $v_B = 3.70 \text{ m/s}$

a.) v_A' ; v_B'

b.) Δp ?

a.) from 25:

$$v_A' = \frac{v_A \left(\frac{m_A}{m_B} - 1 \right) + 2v_B}{\left(\frac{m_A}{m_B} + 1 \right)} = \boxed{3.62 \text{ m/s}}$$

$$v_B' = \boxed{4.42 \text{ m/s}}$$

b.) $\Delta p_A = p_A - p_i = m_A (v_A' - v_A) = \boxed{-396 \frac{\text{kg} \cdot \text{m}}{\text{s}}}$

$$\Delta p_B = \boxed{+396 \frac{\text{kg} \cdot \text{m}}{\text{s}}}$$